

Chapter 17 – Probability Models

1. Bernoulli.

- a) These are not Bernoulli trials. The possible outcomes are 1, 2, 3, 4, 5, and 6. There are more than two possible outcomes.
- b) These may be considered Bernoulli trials. There are only two possible outcomes, Type A and not Type A. Assuming the 120 donors are representative of the population, the probability of having Type A blood is 43%. The trials are not independent, because the population is finite, but the 120 donors represent less than 10% of all possible donors.
- c) These are not Bernoulli trials. The probability of getting a heart changes as cards are dealt without replacement.
- d) These are not Bernoulli trials. We are sampling without replacement, so the trials are not independent. Samples without replacement may be considered Bernoulli trials if the sample size is less than 10% of the population, but 500 is more than 10% of 3000.
- e) These may be considered Bernoulli trials. There are only two possible outcomes, sealed properly and not sealed properly. The probability that a package is unsealed is constant, at about 10%, as long as the packages checked are a representative sample of all packages. Finally, the trials are not independent, since the total number of packages is finite, but the 24 packages checked probably represent less than 10% of the packages.

2. Bernoulli 2.

- a) These may be considered Bernoulli trials. There are only two possible outcomes, getting a 6 and not getting a 6. The probability of getting a 6 is constant at $1/6$. The rolls are independent of one another, since the outcome of one die roll doesn't affect the other rolls.
- b) These are not Bernoulli trials. There are more than two possible outcomes for eye color.
- c) These can be considered Bernoulli trials. There are only two possible outcomes, properly attached buttons and improperly attached buttons. As long as the button problem occurs randomly, the probability of a doll having improperly attached buttons is constant at about 3%. The trials are not independent, since the total number of dolls is finite, but 37 dolls is probably less than 10% of all dolls.
- d) These are not Bernoulli trials. The trials are not independent, since the probability of picking a council member with a particular political affiliation changes depending on who has already been picked. The 10% condition is not met, since the sample of size 4 is more than 10% of the population of 19 people.
- e) These may be considered Bernoulli trials. There are only two possible outcomes, cheating and not cheating. Assuming that cheating patterns in this school are similar to the patterns in the nation, the probability that a student has cheated is constant, at 74%. The trials are not independent, since the population of all students is finite, but 481 is less than 10% of all students.

3. Simulating the model.

- a) Answers will vary. A component is the simulation of the picture in one box of cereal. One possible way to model this component is to generate random digits 0-9. Let 0 and 1 represent Tiger Woods and 2-9 a picture of another sports star. Each run will consist of generating random numbers until a 0 or 1 is generated. The response variable will be the number of digits generated until the first 0 or 1.
- b) Answers will vary.
- c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you get a picture of Tiger Woods in the first box. This is the number of trials in which a 0 or 1 was generated first, divided by the total number of trials. Perform similar calculations for the simulated probability that you have to wait until the second box, the third box, etc.
- d) Let X = the number of boxes opened until the first Tiger Woods picture is found.

X	1	2	3	4	5	6	7	8	≥ 9
$P(X)$	0.20	$(0.80)(0.20)$ = 0.16	$(0.80)^2(0.20)$ = 0.128	$(0.80)^3(0.20)$ = 0.1024	0.082	0.066	0.052	0.042	0.168

- e) Answers will vary.

4. Simulation II.

- a) Answers will vary. A component is the simulation of one die roll. One possible way to model this component is to generate random digits 1-6. Let 1 represent getting 1 (the roll you need and let 2-6 represent not getting the roll you need. Each run will consist of generating random numbers until 1 is generated. The response variable will be the number of digits generated until the first 1.
- b) Answers will vary.
- c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you roll a 1 on the first roll. This is the number of trials in which a 1 was generated first divided by the total number of trials. Perform similar calculations for the simulated probability that you have to wait until the second roll, the third roll, etc.
- d) Let X = the number of rolls until the first 1 is rolled.

X	1	2	3	4	5	6	7	8	≥ 9
$P(X)$	$\frac{1}{6}$	$\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)$ ≈ 0.139	$\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)$ ≈ 0.116	$\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)$ ≈ 0.096	$\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$ ≈ 0.80	0.067	0.056	0.047	0.233

- e) Answers will vary.

5. Tiger again.

- a) Answers will vary. A component is the simulation of the picture in one box of cereal. One possible way to model this component is to generate random digits 0-9. Let 0 and 1 represent Tiger Woods and 2-9 a picture of another sports star. Each run will consist of generating five random numbers. The response variable will be the number of 0s and 1s in the five random numbers.
- b) Answers will vary.
- c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that you get no pictures of Tiger Woods in the five boxes. This is the number of trials in which neither 0 nor 1 were generated divided by the total number of trials. Perform similar calculations for the simulated probability that you would get one picture, 2 pictures, etc.
- d) Let X = the number of Tiger Woods pictures in 5 boxes.

X	0	1	2	3	4	5
$P(X)$	$(0.20)^0(0.80)^5$ ≈ 0.33	$\binom{5}{1}(0.20)^1(0.80)^4$ ≈ 0.41	$\binom{5}{2}(0.20)^2(0.80)^3$ ≈ 0.20	$\binom{5}{3}(0.20)^3(0.80)^2$ ≈ 0.05	$\binom{5}{4}(0.20)^4(0.80)^1$ ≈ 0.01	$(0.20)^5(0.80)^0$ ≈ 0.0

- e) Answers will vary.

6. Seatbelts.

- a) Answers will vary. A component is the simulation of one driver in a car. One possible way to model this component is to generate pairs of random digits 00-99. Let 01-75 represent a driver wearing his or her seatbelt and let 76-99 and 00 represent a driver not wearing his or her seatbelt. Each run will consist of generating five pairs of random digits. The response variable will be the number of pairs of digits that are 00-75.
- b) Answers will vary.
- c) Answers will vary. To construct your simulated probability model, start by calculating the simulated probability that none of the five drivers are wearing seatbelts. This is the number of trials in which no pairs of digits were 00-75, divided by the total number of trials. Perform similar calculations for the simulated probability that one driver is wearing his or her seatbelt, two drivers, etc.
- d) Let X = the number of drivers wearing seatbelts in 5 cars.

X	0	1	2	3	4	5
$P(X)$	$(0.75)^0(0.25)^5$ ≈ 0.0	$\binom{5}{1}(0.75)^1(0.25)^4$ ≈ 0.01	$\binom{5}{2}(0.75)^2(0.25)^3$ ≈ 0.09	$\binom{5}{3}(0.75)^3(0.25)^2$ ≈ 0.26	$\binom{5}{4}(0.75)^4(0.25)^1$ ≈ 0.40	$(0.75)^5(0.25)^0$ ≈ 0.24

- e) Answers will vary.

7. On time.

These departures cannot be considered Bernoulli trials. Departures from the same airport during a 2-hour period may not be independent. They all might be affected by weather and delays.

8. Lost luggage.

The fate of these bags cannot be considered Bernoulli trials. What happens to 22 pieces of luggage, all checked on the same flight probably aren't independent.

9. Hoops.

The player's shots may be considered Bernoulli trials. There are only two possible outcomes (make or miss), the probability of making a shot is constant (80%), and the shots are independent of one another (making, or missing, a shot does not affect the probability of making the next).

Let X = the number of shots until the first missed shot.

Let Y = the number of shots until the first made shot.

Since these problems deal with shooting until the first miss (or until the first made shot), a geometric model, either $Geom(0.8)$ or $Geom(0.2)$, is appropriate.

- a) Use $Geom(0.2)$. $P(X = 5) = (0.8)^4(0.2) = 0.08192$ (Four shots made, followed by a miss.)
- b) Use $Geom(0.8)$. $P(Y = 4) = (0.2)^3(0.8) = 0.0064$ (Three misses, then a made shot.)
- c) Use $Geom(0.8)$. $P(Y = 1) + P(Y = 2) + P(Y = 3) = (0.8) + (0.2)(0.8) + (0.2)^2(0.8) = 0.992$

10. Chips.

The selection of chips may be considered Bernoulli trials. There are only two possible outcomes (fail testing and pass testing). Provided that the chips selected are a representative sample of all chips, the probability that a chip fails testing is constant at 2%. The trials are not independent, since the population of chips is finite, but we won't need to sample more than 10% of all chips.

Let X = the number of chips required until the first bad chip.

The appropriate model is $Geom(0.02)$.

- a) $P(X = 5) = (0.98)^4(0.02) \approx 0.0184$ (Four good chips, then a bad one.)
- b) $P(1 \leq X \leq 10) = (0.02) + (0.98)(0.02) + (0.98)^2(0.02) + \dots + (0.98)^9(0.02) \approx 0.183$

(Use the geometric model on a calculator or computer for this one!)

11. More hoops.

As determined in a previous exercise, the shots can be considered Bernoulli trials, and since the player is shooting until the first miss, $Geom(0.2)$ is the appropriate model.

$$E(X) = \frac{1}{p} = \frac{1}{0.2} = 5 \text{ shots} \quad \text{The player is expected to take 5 shots until the first miss.}$$

12. Chips ahoy.

As determined in a previous exercise, the selection of chips can be considered Bernoulli trials, and since the company is selecting until the first bad chip, $Geom(0.02)$ is the appropriate model.

$$E(X) = \frac{1}{p} = \frac{1}{0.02} = 50 \text{ chips} \quad \text{The first bad chip is expected to be the 50}^{\text{th}} \text{ chip selected.}$$

13. Customer center operator.

The calls can be considered Bernoulli trials. There are only two possible outcomes, taking the promotion, and not taking the promotion. The probability of success is constant at 5% (50% of the 10% Platinum cardholders.) The trials are not independent, since there are a finite number of cardholders, but this is a major credit card company, so we can assume we are selecting fewer than 10% of all cardholders. Since we are calling people until the first success, the model $Geom(0.05)$ may be used.

$$E(\text{calls}) = \frac{1}{p} = \frac{1}{0.05} = 20 \text{ calls. We expect it to take 20 calls to find the first cardholder to take the double miles promotion.}$$

14. Cold calls.

The donor contacts can be considered Bernoulli trials. There are only two possible outcomes, giving \$100 or more, and not giving \$100 or more. The probability of success is constant at 1% (5% of the 20% of donors who will make a donation.) The trials are not independent, since there are a finite number of potential donors, but we will assume that she is contacting less than 10% of all possible donors. Since we are contacting people until the first success, the model $Geom(0.01)$ may be used.

$$E(\text{contacts}) = \frac{1}{p} = \frac{1}{0.01} = 100 \text{ contacts. We expect that Justine will have to contact 100 potential donors to find a \$100 donor.}$$

15. Blood.

These may be considered Bernoulli trials. There are only two possible outcomes, Type AB and not Type AB. Provided that the donors are representative of the population, the probability of having Type AB blood is constant at 4%. The trials are not independent, since the population is finite, but we are selecting fewer than 10% of all potential donors. Since we are selecting people until the first success, the model $Geom(0.04)$ may be used.

Let X = the number of donors until the first Type AB donor is found.

a) $E(X) = \frac{1}{p} = \frac{1}{0.04} = 25 \text{ people} \quad \text{We expect the 25}^{\text{th}} \text{ person to be the first Type AB donor.}$

b)

$$\begin{aligned} &P(\text{a Type AB donor among the first 5 people checked}) \\ &= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \\ &= (0.04) + (0.96)(0.04) + (0.96)^2(0.04) + (0.96)^3(0.04) + (0.96)^4(0.04) \approx 0.185 \end{aligned}$$

c)

$$\begin{aligned}
 &P(\text{a Type AB donor among the first 6 people checked}) \\
 &= P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) \\
 &= (0.04) + (0.96)(0.04) + (0.96)^2(0.04) + (0.96)^3(0.04) + (0.96)^4(0.04) + (0.96)^5(0.04) \approx 0.217
 \end{aligned}$$

- d) $P(\text{no Type AB donor before the 10th person checked}) = P(X > 9) = (0.96)^9 \approx 0.693$
 This one is a bit tricky. There is no implication that we actually find a donor on the 10th trial. We only care that nine trials passed with no Type AB donor.

16. Colorblindness.

These may be considered Bernoulli trials. There are only two possible outcomes, colorblind and not colorblind. As long as the men selected are representative of the population of all men, the probability of being colorblind is constant at about 8%. Trials are not independent, since the population is finite, but we won't be sampling more than 10% of the population.

Let X = the number of people checked until the first colorblind man is found.

Since we are selecting people until the first success, the model $Geom(0.08)$, may be used.

- a) $E(X) = \frac{1}{p} = \frac{1}{0.08} = 12.5$ people. We expect to examine 12.5 people until finding the first colorblind person.

- b) $P(\text{no colorblind men among the first 4}) = P(X > 4) = (0.92)^4 \approx 0.716$

- c) $P(\text{first colorblind man is the sixth man checked}) = P(X = 6) = (0.92)^5(0.08) \approx 0.0527$

d)

$$\begin{aligned}
 &P(\text{she finds a colorblind man before the tenth man}) \\
 &= P(1 \leq X \leq 9) \\
 &= (0.08) + (0.92)(0.08) + (0.92)^2(0.08) + \dots + (0.92)^8(0.08) \approx 0.528
 \end{aligned}$$

(Use the geometric model on a calculator or computer for this one!)

17. Lefties.

These may be considered Bernoulli trials. There are only two possible outcomes, left-handed and not left-handed. Since people are selected at random, the probability of being left-handed is constant at about 13%. The trials are not independent, since the population is finite, but a sample of 5 people is certainly fewer than 10% of all people.

Let X = the number of people checked until the first lefty is discovered.

Let Y = the number of lefties among $n = 5$.

- a) Use $Geom(0.13)$.

$$P(\text{first lefty is the fifth person}) = P(X = 5) = (0.87)^4(0.13) \approx 0.0745$$

b) Use $\text{Binom}(5, 0.13)$.

$$\begin{aligned} P(\text{some lefties among the 5 people}) &= 1 - P(\text{no lefties among the first 5 people}) \\ &= 1 - P(Y = 0) \\ &= 1 - \binom{5}{0}(0.13)^0(0.87)^5 \\ &\approx 0.502 \end{aligned}$$

c) Use $\text{Geom}(0.13)$.

$$P(\text{first lefty is second or third person}) = P(X = 2) + P(X = 3) = (0.87)(0.13) + (0.87)^2(0.13) \approx 0.211$$

d) Use $\text{Binom}(5, 0.13)$.

$$P(\text{exactly 3 lefties in the group}) = P(Y = 3) = \binom{5}{3}(0.13)^3(0.87)^2 \approx 0.0166$$

e) Use $\text{Binom}(5, 0.13)$.

$$\begin{aligned} P(\text{at least 3 lefties in the group}) &= P(Y = 3) + P(Y = 4) + P(Y = 5) \\ &= \binom{5}{3}(0.13)^3(0.87)^2 + \binom{5}{4}(0.13)^4(0.87)^1 + \binom{5}{5}(0.13)^5(0.87)^0 \\ &\approx 0.0179 \end{aligned}$$

f) Use $\text{Binom}(5, 0.13)$.

$$\begin{aligned} P(\text{at most 3 lefties in the group}) &= P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3) \\ &= \binom{5}{0}(0.13)^0(0.87)^5 + \binom{5}{1}(0.13)^1(0.87)^4 \\ &\quad + \binom{5}{2}(0.13)^2(0.87)^3 + \binom{5}{3}(0.13)^3(0.87)^2 \\ &\approx 0.9987 \end{aligned}$$

18. Arrows.

These may be considered Bernoulli trials. There are only two possible outcomes, hitting the bull's-eye and not hitting the bull's-eye. The probability of hitting the bull's-eye is given, $p = 0.80$. The shots are assumed to be independent.

Let X = the number of shots until the first bull's-eye.

Let Y = the number of bull's-eyes in $n = 6$ shots.

a) Use $\text{Geom}(0.80)$.

$$P(\text{first bull's-eye is on the third shot}) = P(X = 3) = (0.20)^2(0.80) \approx 0.032$$

b) Use $\text{Binom}(6, 0.80)$.

$$\begin{aligned} P(\text{at least one miss out of 6 shots}) &= 1 - P(6 \text{ out of 6 hits}) \\ &= 1 - P(Y = 6) \\ &= 1 - \binom{6}{6}(0.80)^6(0.20)^0 \\ &\approx 0.738 \end{aligned}$$

248 Part IV Randomness and Probability

- c) Use
- Geom*
- (0.80).

$$P(\text{first hit on fourth or fifth shot}) = P(X = 4) + P(X = 5) = (0.20)^3(0.80) + (0.20)^4(0.80) = 0.00768$$

- d) Use
- Binom*
- (6,0.80).

$$\begin{aligned} P(\text{exactly four hits}) &= P(Y = 4) \\ &= \binom{6}{4}(0.80)^4(0.20)^2 \\ &\approx 0.246 \end{aligned}$$

- e) Use
- Binom*
- (6,0.80).

$$\begin{aligned} P(\text{at least four hits}) &= P(Y = 4) + P(Y = 5) + P(Y = 6) \\ &= \binom{6}{4}(0.80)^4(0.20)^2 + \binom{6}{5}(0.80)^5(0.20)^1 + \binom{6}{6}(0.80)^6(0.20)^0 \\ &\approx 0.901 \end{aligned}$$

- f) Use
- Binom*
- (6,0.80).

$$\begin{aligned} P(\text{at most four hits}) &= P(Y = 0) + P(Y = 1) + P(Y = 2) + P(Y = 3) + P(Y = 4) \\ &= \binom{6}{0}(0.80)^0(0.20)^6 + \binom{6}{1}(0.80)^1(0.20)^5 + \binom{6}{2}(0.80)^2(0.20)^4 \\ &\quad + \binom{6}{3}(0.80)^3(0.20)^3 + \binom{6}{4}(0.80)^4(0.20)^2 \\ &\approx 0.345 \end{aligned}$$

19. Lefties redux.

- a) In a previous exercise, we determined that the selection of lefties could be considered Bernoulli trials. Since our group consists of 5 people, use
- Binom*
- (5,0.13).

Let Y = the number of lefties among $n = 5$.

$$E(Y) = np = 5(0.13) = 0.65 \text{ lefties}$$

- b)
- $SD(Y) = \sqrt{npq} = \sqrt{5(0.13)(0.87)} \approx 0.75$
- lefties

- c) Use
- Geom*
- (0.13). Let
- X
- = the number of people checked until the first lefty is discovered.

$$E(X) = \frac{1}{p} = \frac{1}{0.13} \approx 7.69 \text{ people}$$

20. More arrows.

- a) In a previous exercise, we determined that the shots could be considered Bernoulli trials. Since the archer is shooting 6 arrows, use
- Binom*
- (6,0.80).

Let Y = the number of bull's-eyes in $n = 6$ shots.

$$E(Y) = np = 6(0.80) = 4.8 \text{ bull's-eyes.}$$

- b)
- $SD(Y) = \sqrt{npq} = \sqrt{6(0.80)(0.20)} \approx 0.98$
- bull's-eyes.

- c) Use $Geom(0.80)$. Let X = the number of arrows shot until the first bull's-eye.

$$E(X) = \frac{1}{p} = \frac{1}{0.80} = 1.25 \text{ shots.}$$

21. Still more lefties.

- a) In a previous exercise, we determined that the selection of lefties (and also righties) could be considered Bernoulli trials. Since our group consists of 12 people, and now we are considering the righties, use $Binom(12, 0.87)$.

Let Y = the number of righties among $n = 12$.

$$E(Y) = np = 12(0.87) = 10.44 \text{ righties}$$

$$SD(Y) = \sqrt{npq} = \sqrt{12(0.87)(0.13)} \approx 1.16 \text{ righties}$$

- b) i)

$$\begin{aligned} P(\text{not all righties}) &= 1 - P(\text{all righties}) \\ &= 1 - P(Y = 12) \\ &= 1 - \binom{12}{12} (0.87)^{12} (0.13)^0 \\ &\approx 0.812 \end{aligned}$$

- ii)

$$\begin{aligned} P(\text{no more than 10 righties}) &= P(Y \leq 10) \\ &= P(Y = 0) + P(Y = 1) + P(Y = 2) + \dots + P(Y = 10) \\ &= \binom{12}{0} (0.87)^0 (0.13)^{12} + \binom{12}{1} (0.87)^1 (0.13)^{11} + \dots + \binom{12}{10} (0.87)^{10} (0.13)^2 \\ &\approx 0.475 \end{aligned}$$

- iii)

$$\begin{aligned} P(\text{exactly six of each}) &= P(Y = 6) \\ &= \binom{12}{6} (0.87)^6 (0.13)^6 \\ &\approx 0.00193 \end{aligned}$$

- iv)

$$\begin{aligned} P(\text{majority righties}) &= P(Y \geq 7) \\ &= P(Y = 7) + P(Y = 8) + P(Y = 9) + \dots + P(Y = 12) \\ &= \binom{12}{7} (0.87)^7 (0.13)^5 + \binom{12}{8} (0.87)^8 (0.13)^4 + \dots + \binom{12}{12} (0.87)^{12} (0.13)^0 \\ &\approx 0.998 \end{aligned}$$

22. Still more arrows.

- a) In a previous exercise, we determined that the archer's shots could be considered Bernoulli trials. Since our archer is now shooting 10 arrows, use $\text{Binom}(10, 0.80)$.

Let Y = the number of bull's-eyes hit from $n = 10$ shots.

$$E(Y) = np = 10(0.80) = 8 \text{ bull's-eyes hit.}$$

$$SD(Y) = \sqrt{npq} = \sqrt{10(0.80)(0.20)} \approx 1.26 \text{ bull's-eyes hit.}$$

- b) i)

$$\begin{aligned} P(\text{no misses out of 10 shots}) &= P(\text{all hits out of 10 shots}) \\ &= P(Y = 10) \\ &= \binom{10}{10} (0.80)^{10} (0.20)^0 \\ &\approx 0.107 \end{aligned}$$

- ii)

$$\begin{aligned} P(\text{no more than 8 hits}) &= P(Y \leq 8) \\ &= P(Y = 0) + P(Y = 1) + P(Y = 2) + \dots + P(Y = 8) \\ &= \binom{10}{0} (0.80)^0 (0.20)^{10} + \binom{10}{1} (0.80)^1 (0.20)^9 + \dots + \binom{10}{8} (0.80)^8 (0.20)^2 \\ &\approx 0.624 \end{aligned}$$

- iii)

$$\begin{aligned} P(\text{exactly 8 out of 10 shots}) &= P(Y = 8) \\ &= \binom{10}{8} (0.80)^8 (0.20)^2 \\ &\approx 0.302 \end{aligned}$$

- iv)

$$\begin{aligned} P(\text{more hits than misses}) &= P(Y \geq 6) \\ &= P(Y = 6) + P(Y = 7) + \dots + P(Y = 10) \\ &= \binom{10}{6} (0.80)^6 (0.20)^4 + \binom{10}{7} (0.80)^7 (0.20)^3 + \dots + \binom{10}{10} (0.80)^{10} (0.20)^0 \\ &\approx 0.967 \end{aligned}$$

23. Vision.

The vision tests can be considered Bernoulli trials. There are only two possible outcomes, nearsighted or not. The probability of any child being nearsighted is given as $p = 0.12$. Finally, since the population of children is finite, the trials are not independent. However, 169 is certainly less than 10% of all children, and we will assume that the children in this district are representative of all children in relation to nearsightedness. Use $\text{Binom}(169, 0.12)$.

$$\mu = E(\text{nearsighted}) = np = 169(0.12) = 20.28 \text{ children.}$$

$$\sigma = SD(\text{nearsighted}) = \sqrt{npq} = \sqrt{169(0.12)(0.88)} \approx 4.22 \text{ children.}$$

24. International students.

The students can be considered Bernoulli trials. There are only two possible outcomes, international or not. The probability of any freshmen being an international student is given as $p = 0.06$. Finally, since the population of freshmen is finite, the trials are not independent. However, 40 is likely to be less than 10% of all students, and we are told that the freshmen in this college are randomly assigned to housing.

Use $\text{Binom}(40, 0.06)$.

$$\mu = E(\text{international}) = np = 40(0.06) = 2.4 \text{ students.}$$

$$\sigma = SD(\text{nearsighted}) = \sqrt{npq} = \sqrt{40(0.06)(0.94)} \approx 1.5 \text{ students.}$$

25. Tennis, anyone?

The first serves can be considered Bernoulli trials. There are only two possible outcomes, successful and unsuccessful. The probability of any first serve being good is given as $p = 0.70$. Finally, we are assuming that each serve is independent of the others. Since she is serving 6 times, use $\text{Binom}(6, 0.70)$.

Let X = the number of successful serves in $n = 6$ first serves.

- a) $P(\text{all six serves in}) = P(X = 6)$ b) $P(\text{exactly four serves in}) = P(X = 4)$
- $$= \binom{6}{6}(0.70)^6(0.30)^0$$
- $$= \binom{6}{4}(0.70)^4(0.30)^2$$
- $$\approx 0.118$$
- $$\approx 0.324$$
- c) $P(\text{at least four serves in}) = P(X = 4) + P(X = 5) + P(X = 6)$
- $$= \binom{6}{4}(0.70)^4(0.30)^2 + \binom{6}{5}(0.70)^5(0.30)^1 + \binom{6}{6}(0.70)^6(0.30)^0$$
- $$\approx 0.744$$
- d) $P(\text{no more than four serves in}) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$
- $$= \binom{6}{0}(0.70)^0(0.30)^6 + \binom{6}{1}(0.70)^1(0.30)^5 + \binom{6}{2}(0.70)^2(0.30)^4$$
- $$+ \binom{6}{3}(0.70)^3(0.30)^3 + \binom{6}{4}(0.70)^4(0.30)^2$$
- $$\approx 0.580$$

26. Frogs.

The frog examinations can be considered Bernoulli trials. There are only two possible outcomes, having the trait and not having the trait. If the frequency of the trait has not changed, and the biologist collects a representative sample of frogs, then the probability of a frog having the trait is constant, at $p = 0.125$. The trials are not independent since the population of frogs is finite, but 12 frogs is fewer than 10% of all frogs. Since the biologist is collecting 12 frogs, use $\text{Binom}(12, 0.125)$.

Let X = the number of frogs with the trait, from $n = 12$ frogs.

252 Part IV Randomness and Probability

a)

$$\begin{aligned}
 P(\text{no frogs have the trait}) &= P(X = 0) \\
 &= \binom{12}{0} (0.125)^0 (0.875)^{12} \\
 &\approx 0.201
 \end{aligned}$$

b)

$$\begin{aligned}
 P(\text{at least two frogs}) &= P(X \geq 2) \\
 &= P(X = 2) + P(X = 3) + \dots + P(X = 12) \\
 &= \binom{12}{2} (0.125)^2 (0.875)^{10} + \binom{12}{3} (0.125)^3 (0.875)^9 + \dots + \binom{12}{12} (0.125)^{12} (0.875)^0 \\
 &\approx 0.453
 \end{aligned}$$

c)

$$\begin{aligned}
 P(\text{three or four frogs have trait}) &= P(X = 3) + P(X = 4) \\
 &= \binom{12}{3} (0.125)^3 (0.875)^9 + \binom{12}{4} (0.125)^4 (0.875)^8 \\
 &\approx 0.171
 \end{aligned}$$

d)

$$\begin{aligned}
 P(\text{no more than four}) &= P(X \leq 4) = P(X = 0) + P(X = 1) + \dots + P(X = 4) \\
 &= \binom{12}{0} (0.125)^0 (0.875)^{12} + \binom{12}{1} (0.125)^1 (0.875)^{11} + \dots + \binom{12}{4} (0.125)^4 (0.875)^8 \\
 &\approx 0.989
 \end{aligned}$$

27. And more tennis.

The first serves can be considered Bernoulli trials. There are only two possible outcomes, successful and unsuccessful. The probability of any first serve being good is given as $p = 0.70$. Finally, we are assuming that each serve is independent of the others. Since she is serving 80 times, use $\text{Binom}(80, 0.70)$.

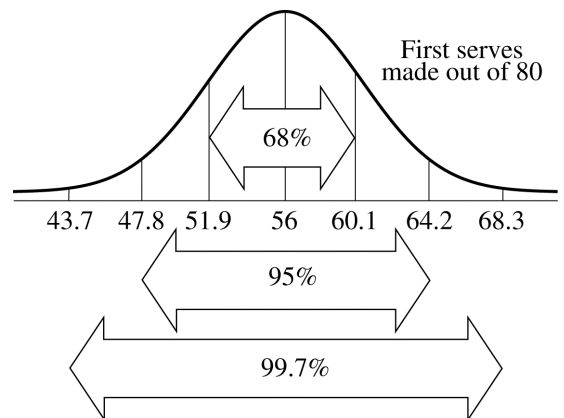
Let X = the number of successful serves in $n = 80$ first serves.

a) $E(X) = np = 80(0.70) = 56$ first serves in.

$$SD(X) = \sqrt{npq} = \sqrt{80(0.70)(0.30)} \approx 4.10 \text{ first serves in.}$$

b) Since $np = 56$ and $nq = 24$ are both greater than 10, $\text{Binom}(80, 0.70)$ may be approximated by the Normal model, $N(56, 4.10)$.

c) According to the Normal model, in matches with 80 serves, she is expected to make between 51.9 and 60.1 first serves approximately 68% of the time, between 47.8 and 64.2 first serves approximately 95% of the time, and between 43.7 and 68.3 first serves approximately 99.7% of the time.



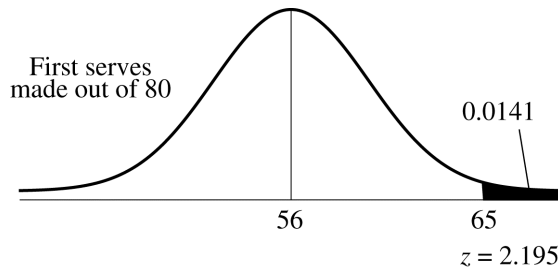
d) Using *Binom*(80, 0.70):

$$\begin{aligned}
 P(\text{at least 65 first serves}) &= P(X \geq 65) \\
 &= P(X = 65) + P(X = 66) + \dots + P(X = 80) \\
 &= \binom{80}{65}(0.70)^{65}(0.30)^{15} + \binom{80}{66}(0.70)^{66}(0.30)^{14} + \dots + \binom{80}{80}(0.70)^{80}(0.30)^0 \\
 &\approx 0.0161
 \end{aligned}$$

According to the Binomial model, the probability that she makes at least 65 first serves out of 80 is approximately 0.0161.

Using *N*(56, 4.10):

$$\begin{aligned}
 z &= \frac{x - \mu}{\sigma} \\
 z &= \frac{65 - 56}{4.10} \\
 z &\approx 2.195
 \end{aligned}$$



According to the Normal model, the probability that she makes at least 65 first serves out of 80 is approximately 0.0141.

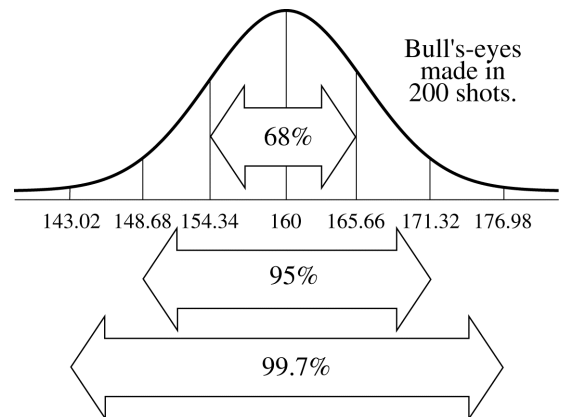
$$P(X \geq 65) \approx P(z > 2.195) \approx 0.0141$$

28. More arrows.

These may be considered Bernoulli trials. There are only two possible outcomes, hitting the bull's-eye and not hitting the bull's-eye. The probability of hitting the bull's-eye is given, $p = 0.80$. The shots are assumed to be independent. Since she will be shooting 200 arrows, use *Binom*(200, 0.80).

Let Y = the number of bull's-eyes in $n = 200$ shots.

- a) $E(Y) = np = 200(0.80) = 160$ bull's-eyes.
 $SD(Y) = \sqrt{npq} = \sqrt{200(0.80)(0.20)} \approx 5.66$ bull's-eyes.
- b) Since $np = 160$ and $nq = 40$ are both greater than 10, *Binom*(200,0.80) may be approximated by the Normal model, *N*(160, 5.66).
- c) According to the Normal model, in matches with 200 arrows, she is expected to get between 154.34 and 165.66 bull's-eyes approximately 68% of the time, between 148.68 and 171.32 bull's-eyes approximately 95% of the time, and between 143.02 and 176.98 bull's-eyes approximately 99.7% of the time.



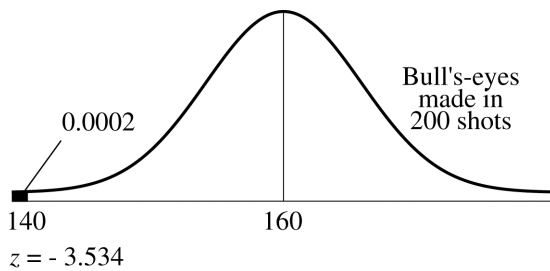
d) Using *Binom*(200, 0.80):

$$\begin{aligned}
 P(\text{at most 140 hits}) &= P(Y \leq 140) \\
 &= P(Y = 0) + P(Y = 1) + \dots + P(Y = 140) \\
 &= \binom{200}{0}(0.80)^0(0.20)^{200} + \binom{200}{1}(0.80)^1(0.20)^{199} + \dots + \binom{200}{140}(0.80)^{140}(0.20)^{60} \\
 &\approx 0.0005
 \end{aligned}$$

According to the Binomial model, the probability that she makes at most 140 bull's-eyes out of 200 is approximately 0.0005.

Using *N*(160, 5.66):

$$\begin{aligned}
 z &= \frac{y - \mu}{\sigma} \\
 z &= \frac{140 - 160}{5.66} \\
 z &\approx -3.534
 \end{aligned}$$



According to the Normal model, the probability that she hits at most 140 bull's-eyes out of 200 is approximately 0.0002.

$P(Y \leq 140) \approx P(z < -3.534) \approx 0.0002$. Using either model, it is apparent that it is very unlikely that the archer would hit only 140 bull's-eyes out of 200.

29. Apples.

a) A binomial model and a normal model are both appropriate for modeling the number of cider apples that may come from the tree.

Let X = the number of cider apples found in the $n = 300$ apples from the tree.

The quality of the apples may be considered Bernoulli trials. There are only two possible outcomes, cider apple or not a cider apple. The probability that an apple must be used for a cider apple is constant, given as $p = 0.06$. The trials are not independent, since the population of apples is finite, but the apples on the tree are undoubtedly less than 10% of all the apples that the farmer has ever produced, so model with *Binom*(300, 0.06).

$$E(X) = np = 300(0.06) = 18 \text{ cider apples.}$$

$$SD(X) = \sqrt{npq} = \sqrt{300(0.06)(0.94)} \approx 4.11 \text{ cider apples.}$$

Since $np = 18$ and $nq = 282$ are both greater than 10, *Binom*(300, 0.06) may be approximated by the Normal model, *N*(18, 4.11).

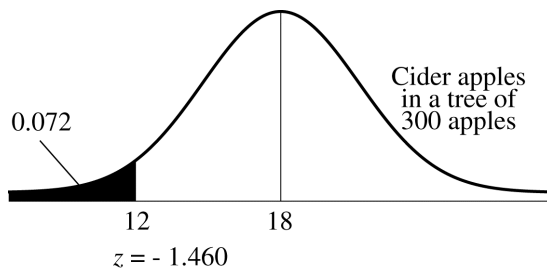
b) Using *Binom*(300, 0.06):

$$\begin{aligned}
 P(\text{at most 12 cider apples}) &= P(X \leq 12) \\
 &= P(X = 0) + \dots + P(X = 12) \\
 &= \binom{300}{0}(0.06)^0(0.94)^{300} + \dots + \binom{300}{12}(0.06)^{12}(0.94)^{282} \\
 &\approx 0.085
 \end{aligned}$$

According to the Binomial model, the probability that no more than 12 cider apples come from the tree is approximately 0.085.

Using *N*(18, 4.11):

$$\begin{aligned}
 z &= \frac{x - \mu}{\sigma} \\
 z &= \frac{12 - 18}{4.11} \\
 z &= -1.460
 \end{aligned}$$



$$P(X \leq 12) \approx P(z < -1.460) \approx 0.072$$

According to the Normal model, the probability that no more than 12 apples out of 300 are cider apples is approximately 0.072.

- c) It is extremely unlikely that the tree will bear more than 50 cider apples. Using the Normal model, *N*(18, 4.11), 50 cider apples is about 7.8 standard deviations above the mean.

30. Frogs, part II.

The frog examinations can be considered Bernoulli trials. There are only two possible outcomes, having the trait and not having the trait. If the frequency of the trait has not changed, and the biologist collects a representative sample of frogs, then the probability of a frog having the trait is constant, at $p = 0.125$. The trials are not independent since the population of frogs is finite, but 150 frogs is fewer than 10% of all frogs. Since the biologist is collecting 150 frogs, use *Binom*(150, 0.125).

Let X = the number of frogs with the trait, from $n = 150$ frogs.

- a) $E(X) = np = 150(0.125) = 18.75$ frogs. $SD(X) = \sqrt{npq} = \sqrt{150(0.125)(0.875)} \approx 4.05$ frogs.
- b) Since $np = 18.75$ and $nq = 131.25$ are both greater than 10, *Binom*(150, 0.125) may be approximated by the Normal model, *N*(18.75, 4.05).
- c) **Using *Binom*(150, 0.125):**

$$\begin{aligned}
 P(\text{at least 22 frogs}) &= P(X \geq 22) \\
 &= P(X = 22) + \dots + P(X = 150) \\
 &= \binom{150}{22}(0.125)^{22}(0.875)^{128} + \dots + \binom{150}{150}(0.125)^{150}(0.875)^0 \\
 &\approx 0.2433
 \end{aligned}$$

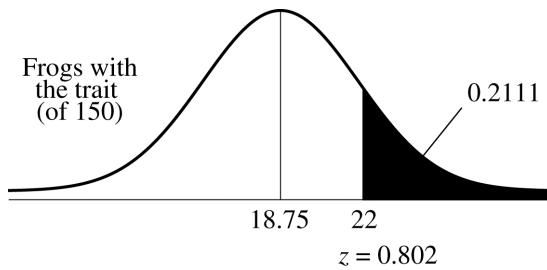
According to the Binomial model, the probability that at least 22 frogs out of 150 have the trait is approximately 0.2433.

Using $N(18.75, 4.05)$:

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{22 - 18.75}{4.05}$$

$$z \approx 0.802$$



$$P(X \geq 22) \approx P(z > 0.802) \approx 0.2111$$

According to the Normal model, the probability that at least 22 frogs out of 150 have the trait is approximately 0.2111.

Using either model, the probability that the biologist discovers 22 of 150 frogs with the trait simply as a result of natural variability is quite high. This doesn't prove that the trait has become more common.

31. Lefties again.

Let X = the number of righties among a class of $n = 188$ students.

Using $\text{Binom}(188, 0.87)$:

These may be considered Bernoulli trials. There are only two possible outcomes, right-handed and not right-handed. The probability of being right-handed is assumed to be constant at about 87%. The trials are not independent, since the population is finite, but a sample of 188 students is certainly fewer than 10% of all people. Therefore, the number of righties in a class of 188 students may be modeled by $\text{Binom}(188, 0.87)$.

If there are 171 or more righties in the class, some righties have to use a left-handed desk.

$$P(\text{at least 171 righties}) = P(X \geq 171)$$

$$= P(X = 171) + \dots + P(X = 188)$$

$$= \binom{188}{171} (0.87)^{171} (0.13)^{17} + \dots + \binom{188}{188} (0.87)^{188} (0.13)^0$$

$$\approx 0.061$$

According to the binomial model, the probability that a right-handed student has to use a left-handed desk is approximately 0.061.

Using $N(163.56, 4.61)$:

$$E(X) = np = 188(0.87) = 163.56 \text{ righties.}$$

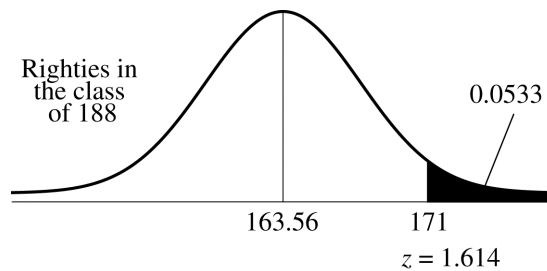
$$SD(X) = \sqrt{npq} = \sqrt{188(0.87)(0.13)} \approx 4.61 \text{ righties.}$$

Since $np = 163.56$ and $nq = 24.44$ are both greater than 10, $\text{Binom}(188, 0.87)$ may be approximated by the Normal model, $N(163.56, 4.61)$.

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{171 - 163.56}{4.61}$$

$$z \approx 1.614$$



$$P(X \geq 171) \approx P(z > 1.614) \approx 0.053$$

According to the Normal model, the probability that there are at least 171 righties in the class of 188 is approximately 0.0533.

32. No-shows.

Let X = the number of passengers that show up for the flight of $n = 275$ passengers.

Using *Binom*(275, 0.95):

These may be considered Bernoulli trials. There are only two possible outcomes, showing up and not showing up. The airlines believe the probability of showing up is constant at about 95%. The trials are not independent, since the population is finite, but a sample of 275 passengers is certainly fewer than 10% of all passengers. Therefore, the number of passengers who show up for a flight of 275 may be modeled by *Binom*(275, 0.95).

If 266 or more passengers show up, someone has to get bumped off the flight.

$$P(\text{at least 266 passengers}) = P(X \geq 266)$$

$$= P(X = 266) + \dots + P(X = 275)$$

$$= \binom{275}{266} (0.95)^{266} (0.05)^9 + \dots + \binom{275}{275} (0.95)^{275} (0.05)^0$$

$$\approx 0.116$$

According to the binomial model, the probability someone on the flight must be bumped is approximately 0.116.

Using *N*(261.25, 3.61):

$$E(X) = np = 275(0.95) = 261.25 \text{ passengers.}$$

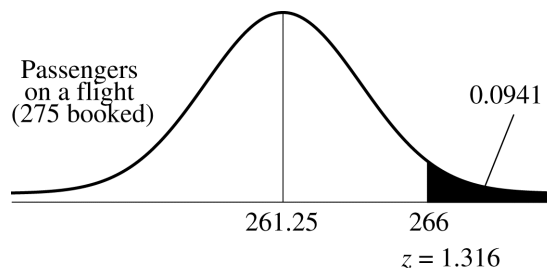
$$SD(X) = \sqrt{npq} = \sqrt{275(0.95)(0.05)} \approx 3.61 \text{ passengers.}$$

Since $np = 261.25$ and $nq = 13.75$ are both greater than 10, *Binom*(275, 0.95) may be approximated by the Normal model, *N*(261.25, 3.61).

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{266 - 261.25}{3.61}$$

$$z \approx 1.316$$



$$P(X \geq 266) \approx P(z > 1.316) \approx 0.0941$$

According to the Normal model, the probability that at least 266 passengers show up is approximately 0.0941.

33. Annoying phone calls.

Let X = the number of sales made after making $n = 200$ calls.

Using $Binom(200, 0.12)$:

These may be considered Bernoulli trials. There are only two possible outcomes, making a sale and not making a sale. The telemarketer was told that the probability of making a sale is constant at about $p = 0.12$. The trials are not independent, since the population is finite, but 200 calls is fewer than 10% of all calls. Therefore, the number of sales made after making 200 calls may be modeled by $Binom(200, 0.12)$.

$$\begin{aligned} P(\text{at most } 10) &= P(X \leq 10) \\ &= P(X = 0) + \dots + P(X = 10) \\ &= \binom{200}{0}(0.12)^0(0.88)^{200} + \dots + \binom{200}{10}(0.12)^{10}(0.88)^{190} \\ &\approx 0.0006 \end{aligned}$$

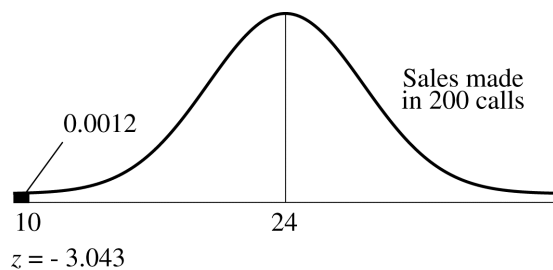
According to the Binomial model, the probability that the telemarketer would make at most 10 sales is approximately 0.0006.

Using $N(24, 4.60)$:

$$\begin{aligned} E(X) &= np = 200(0.12) = 24 \text{ sales.} \\ SD(X) &= \sqrt{npq} = \sqrt{200(0.12)(0.88)} \approx 4.60 \text{ sales.} \end{aligned}$$

Since $np = 24$ and $nq = 176$ are both greater than 10, $Binom(200, 0.12)$ may be approximated by the Normal model, $N(24, 4.60)$.

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ z &= \frac{10 - 24}{4.60} \\ z &\approx -3.043 \end{aligned}$$



$$P(X \leq 10) \approx P(z < -3.043) \approx 0.0012$$

According to the Normal model, the probability that the telemarketer would make at most 10 sales is approximately 0.0012.

Since the probability that the telemarketer made 10 sales, given that the 12% of calls result in sales is so low, it is likely that he was misled about the true success rate.

34. The euro.

Let X = the number of heads after spinning a Belgian euro $n = 250$ times.

Using $Binom(250, 0.5)$:

These may be considered Bernoulli trials. There are only two possible outcomes, heads and tails. The probability that a fair Belgian euro lands heads is $p = 0.5$. The trials are independent, since the outcome of a spin does not affect other spins.

Therefore, $Binom(250, 0.5)$ may be used to model the number of heads after spinning a Belgian euro 250 times.

$$\begin{aligned}
 P(\text{at least } 140) &= P(X \geq 140) \\
 &= P(X = 140) + \dots + P(X = 250) \\
 &= \binom{250}{140} (0.5)^{140} (0.5)^{110} + \dots + \binom{250}{250} (0.5)^{250} (0.5)^0 \\
 &\approx 0.0332
 \end{aligned}$$

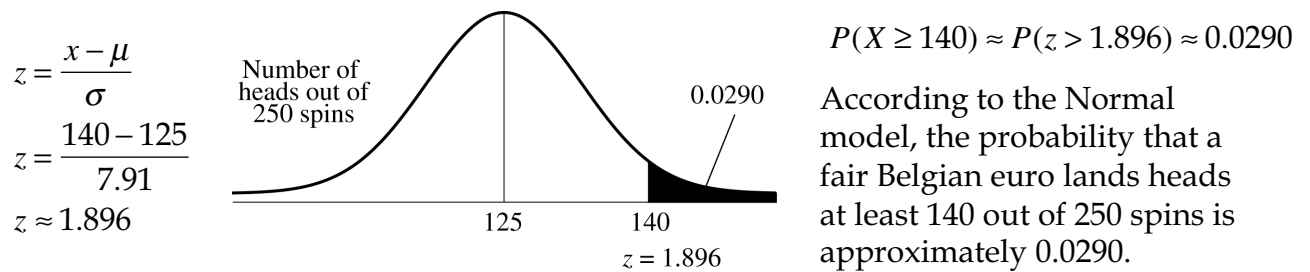
According to the Binomial model, the probability that a fair Belgian euro comes up heads at least 140 times is 0.0332.

Using $N(125, 7.91)$:

$$E(X) = np = 250(0.05) = 125 \text{ heads.}$$

$$SD(X) = \sqrt{npq} = \sqrt{250(0.5)(0.5)} \approx 7.91 \text{ heads.}$$

Since $np = 125$ and $nq = 125$ are both greater than 10, $\text{Binom}(250, 0.5)$ may be approximated by the Normal model, $N(125, 7.91)$.



Since the probability that a fair Belgian euro lands heads at least 140 out of 250 spins is low, it is unlikely that the euro spins fairly. However, the probability is not extremely low, and we aren't sure of the source of the data, so it might be a good idea to spin it some more.

35. Seatbelts II.

These stops may be considered Bernoulli trials. There are only two possible outcomes, belted or not belted. Police estimate that the probability that a driver is buckled is 80%. (The probability of not being buckled is therefore 20%.) Provided the drivers stopped are representative of all drivers, we can consider the probability constant. The trials are not independent, since the population of drivers is finite, but the police will not stop more than 10% of all drivers.

- a) Let X = the number of cars stopped before finding a driver whose seat belt is not buckled. Use $\text{Geom}(0.2)$ to model the situation.

$$E(X) = \frac{1}{p} = \frac{1}{0.2} = 5 \text{ cars.}$$

- b) $P(\text{First unbelted driver is in the sixth car}) = P(X = 6) = (0.8)^5 (0.2) \approx 0.066$
- c) $P(\text{The first ten drivers are wearing seatbelts}) = (0.8)^{10} \approx .107$

260 Part IV Randomness and Probability

- d) Let Y = the number of drivers wearing their seatbelts in 30 cars. Use $Binom(30, 0.8)$.

$$E(Y) = np = 30(0.8) = 24 \text{ drivers.}$$

$$SD(Y) = \sqrt{npq} = \sqrt{30(0.8)(0.2)} \approx 2.19 \text{ drivers.}$$

- e) Let W = the number of drivers not wearing their seatbelts in 120 cars.

Using $Binom(120, 0.2)$:

$$\begin{aligned} P(\text{at least } 20) &= P(W \geq 20) \\ &= P(W = 20) + \dots + P(W = 120) \\ &= \binom{120}{20}(0.2)^{20}(0.8)^{100} + \dots + \binom{120}{120}(0.2)^{120}(0.8)^0 \\ &\approx 0.848 \end{aligned}$$

According to the Binomial model, the probability that at least 20 out of 120 drivers are not wearing their seatbelts is approximately 0.848.

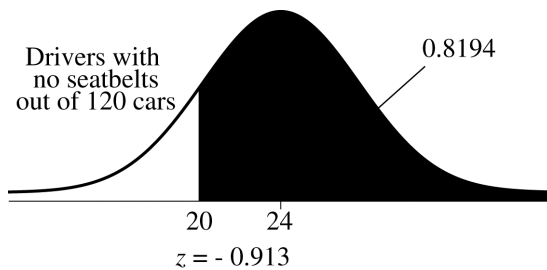
Using $N(24, 4.38)$:

$$E(W) = np = 120(0.2) = 24 \text{ drivers.}$$

$$SD(W) = \sqrt{npq} = \sqrt{120(0.2)(0.8)} \approx 4.38 \text{ drivers.}$$

Since $np = 24$ and $nq = 96$ are both greater than 10, $Binom(120, 0.2)$ may be approximated by the Normal model, $N(24, 4.38)$.

$$\begin{aligned} z &= \frac{w - \mu}{\sigma} \\ z &= \frac{20 - 24}{4.38} \\ z &\approx -0.913 \end{aligned}$$



$$P(W \geq 20) \approx P(z > -0.913) \approx 0.8194$$

According to the Normal model, the probability that at least 20 out of 120 drivers stopped are not wearing their seatbelts is approximately 0.8194.

36. Rickets.

The selection of these children may be considered Bernoulli trials. There are only two possible outcomes, vitamin D deficient or not vitamin D deficient. Recent research indicates that 20% of British children are vitamin D deficient. (The probability of not being vitamin D deficient is therefore 80%.) Provided the students at this school are representative of all British children, we can consider the probability constant. The trials are not independent, since the population of British children is finite, but the children at this school represent fewer than 10% of all British children.

- a) Let X = the number of students tested before finding a student who is vitamin D deficient. Use $Geom(0.2)$ to model the situation.

$$P(\text{First vitamin D deficient child is the eighth one tested}) = P(X = 8) = (0.8)^7(0.2) \approx 0.042$$

- b) $P(\text{The first ten children tested are okay}) = (0.8)^{10} \approx .107$

c) $E(X) = \frac{1}{p} = \frac{1}{0.2} = 5$ kids.

d) Let Y = the number of children who are vitamin D deficient out of 50 children. Use $Binom(50, 0.2)$.

$E(Y) = np = 50(0.2) = 10$ children. $SD(Y) = \sqrt{npq} = \sqrt{50(0.2)(0.8)} \approx 2.83$ children.

e) Using $Binom(320, 0.2)$:

$$\begin{aligned} P(\text{no more than 50 children have the deficiency}) &= P(X \leq 50) \\ &= P(X = 0) + \dots + P(X = 50) \\ &= \binom{320}{0}(0.2)^0(0.8)^{320} + \dots + \binom{320}{50}(0.2)^{50}(0.8)^{270} \\ &\approx 0.027 \end{aligned}$$

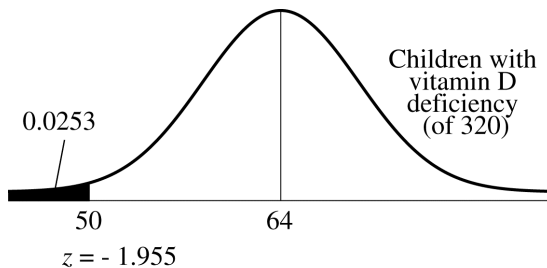
According to the Binomial model, the probability that no more than 50 of the 320 children have the vitamin D deficiency is approximately 0.027.

Using $N(64, 7.16)$:

$E(Y) = np = 320(0.2) = 64$ children. $SD(Y) = \sqrt{npq} = \sqrt{320(0.2)(0.8)} \approx 7.16$ children.

Since $np = 64$ and $nq = 256$ are both greater than 10, $Binom(320, 0.2)$ may be approximated by the Normal model, $N(64, 7.16)$.

$$\begin{aligned} z &= \frac{y - \mu}{\sigma} \\ z &= \frac{50 - 64}{7.16} \\ z &\approx -1.955 \end{aligned}$$



$P(Y \leq 50) \approx P(z < -1.955) \approx 0.0253$

According to the Normal model, the probability that no more than 50 out of 320 children have the vitamin D deficiency is approximately 0.0253.

37. ESP.

Choosing symbols may be considered Bernoulli trials. There are only two possible outcomes, correct or incorrect. Assuming that ESP does not exist, the probability of a correct identification from a randomized deck is constant, at $p = 0.20$. The trials are independent, as long as the deck is shuffled after each attempt. Since 100 trials will be performed, use $Binom(100, 0.2)$.

Let X = the number of symbols identified correctly out of 100 cards.

$E(X) = np = 100(0.2) = 20$ correct identifications.

$SD(X) = \sqrt{npq} = \sqrt{100(0.2)(0.8)} = 4$ correct identifications.

Answers may vary. In order to be convincing, the "mind reader" would have to identify at least 32 out of 100 cards correctly, since 32 is three standard deviations above the mean. Identifying fewer cards than 32 could happen too often, simply due to chance.

38. True-False.

Guessing at answers may be considered Bernoulli trials. There are only two possible outcomes, correct or incorrect. If the student was guessing, the probability of a correct response is constant, at $p = 0.50$. The trials are independent, since the answer to one question should not have any bearing on the answer to the next. Since 50 questions are on the test use $\text{Binom}(500, 0.5)$.

Let X = the number of questions answered correctly out of 50 questions.

$$E(X) = np = 50(0.5) = 25 \text{ correct answers.}$$

$$SD(X) = \sqrt{npq} = \sqrt{50(0.5)(0.5)} \approx 3.54 \text{ correct answers.}$$

Answers may vary. In order to be convincing, the student would have to answer at least 36 out of 50 questions correctly, since 36 is approximately three standard deviations above the mean. Answering fewer than 36 questions correctly could happen too often, simply due to chance.

39. Hot hand.

A streak like this is not unusual. The probability that he makes 4 in a row with a 55% free throw percentage is $(0.55)(0.55)(0.55)(0.55) \approx 0.09$. We can expect this to happen nearly one in ten times for every set of 4 shots that he makes. One out of ten times is not that unusual.

40. New bow.

A streak like this is not unusual. The probability that she makes 6 consecutive bulls-eyes with an 80% bulls-eye percentage is $(0.8)(0.8)(0.8)(0.8)(0.8)(0.8) \approx 0.26$. If she were to shoot several flights of 6 arrows, she is expected to get 6 bulls-eyes about 26% of the time. An event that happens due to chance about one out of four times is not that unusual.

41. Hotter hand.

The shots may be considered Bernoulli trials. There are only two possible outcomes, make or miss. The probability of success is constant at 55%, and the shots are independent of one another. Therefore, we can model this situation with $\text{Binom}(32, 0.55)$.

Let X = the number of free throws made out of 40.

$$E(X) = np = 40(0.55) = 22 \text{ free throws made.}$$

$$SD(X) = \sqrt{npq} = \sqrt{40(0.55)(0.45)} \approx 3.15 \text{ free throws.}$$

Answers may vary. The player's performance seems to have increased. 32 made free throws is $(32 - 22) / 3.15 \approx 3.17$ standard deviations above the mean, an extraordinary feat, unless his free throw percentage has increased. This does NOT mean that the sneakers are responsible for the increase in free throw percentage. Some other variable may account for the increase. The player would need to set up a controlled experiment in order to determine what effect, if any, the sneakers had on his free throw percentage.

42. New bow, again.

The shots may be considered Bernoulli trials. There are only two possible outcomes, hit or miss the bulls-eye. The probability of success is constant at 80%, and the shots are independent of one another. Therefore, we can model this situation with $Binom(50, 0.8)$.

Let X = the number of bulls-eyes hit out of 50.

$$E(X) = np = 50(0.8) = 40 \text{ bulls-eyes hit.}$$

$$SD(X) = \sqrt{npq} = \sqrt{50(0.8)(0.2)} \approx 2.83 \text{ bulls-eyes.}$$

Answers may vary. The archer's performance doesn't seem to have increased. 45 bulls-eyes is $(45 - 40) / 2.83 \approx 1.77$ standard deviations above the mean. This isn't unusual for an archer of her skill level.